# Chapter 6

## Friction

Recall that a contact force *parallel* to the contact surface is called **friction**.

We distinguish now two types: **static friction** and **kinetic friction** (sometimes also called **sliding friction)**. The book refers as well to **rolling friction**, which I am going to consider as another type of kinetic friction

## Maximum force of static friction:

Normal force

$$f_{s,\max} = \mu_s N$$

The constant of proportionality is called the **coefficient of static friction**. Its value depends on the condition and nature of the surfaces.

$$f_s \leq \mu_s N$$

**Why the inequality?**
**What happens if I push a lot or a little against a super heavy stone or my desk?**

A safe is to be moved up a ramp to a height of 1.5 m above the floor.

The mass of the safe is 510 kg, the coefficient of static friction along the incline is $\mu_s = 0.42$, and the coefficient of kinetic friction along the incline is $\mu_k = 0.33$. The ramp forms an angle $\theta = 15°$ above the horizontal.

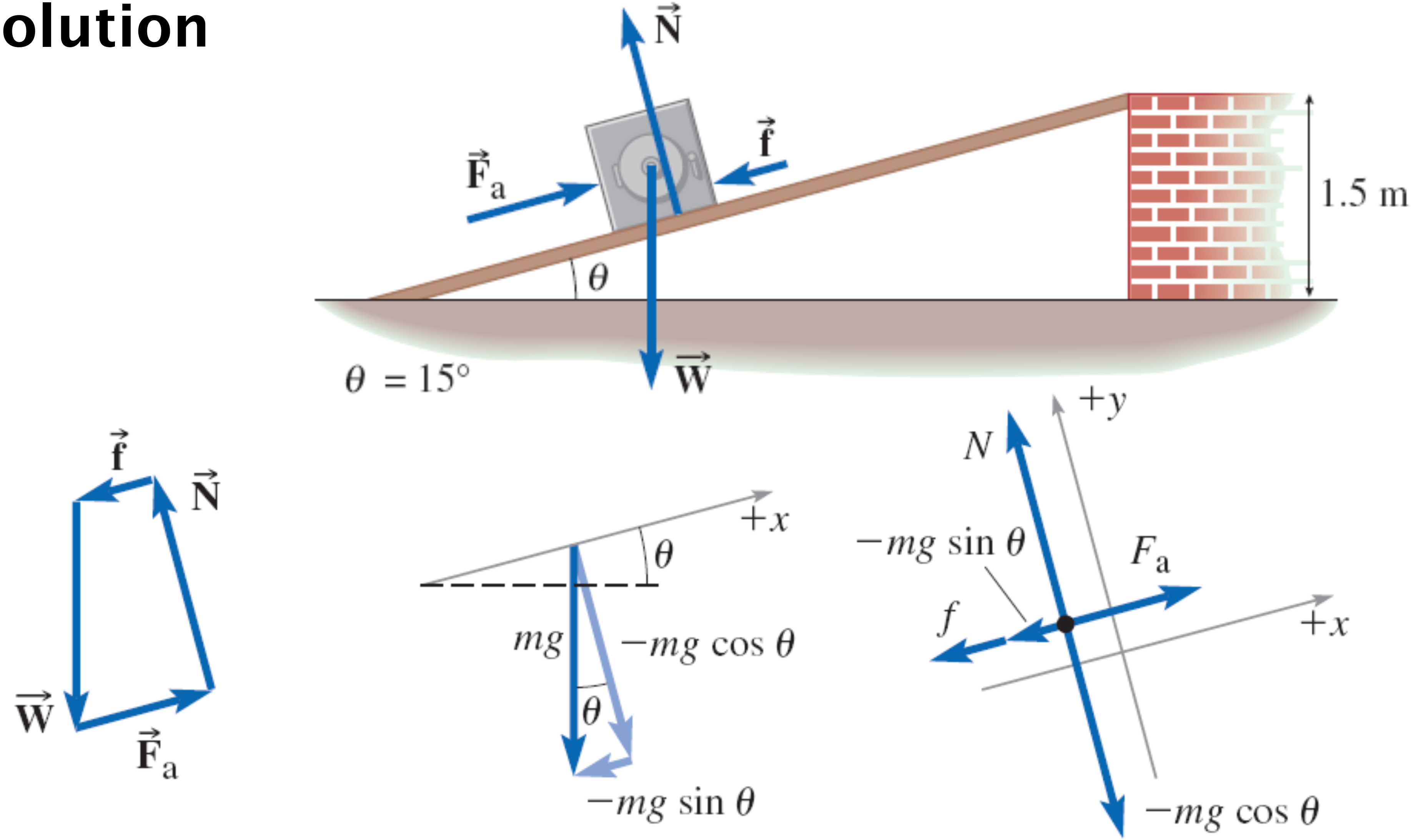
How hard do the movers have to push to start the safe moving up the incline? Assume that they push in a direction parallel to the incline.

## Stepping back first

When the safe *starts* to move, its velocity is changing, so the safe is *not* in equilibrium. Nevertheless, to find the minimum applied force to start the safe moving, we can find the *maximum* applied force for which the safe *remains at rest* —an equilibrium situation.

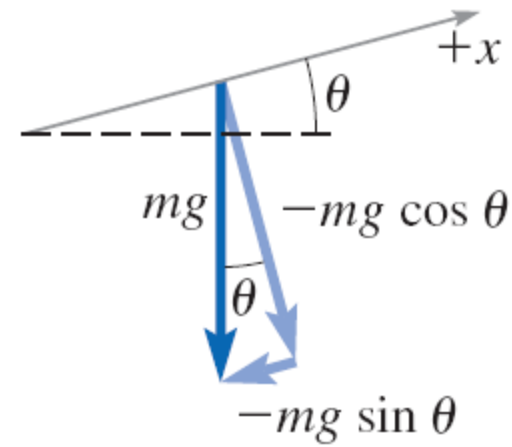
**Let's discuss why this is the right approach**

# Solution




Check angles again (θ = 0 and θ=π)

# Solution



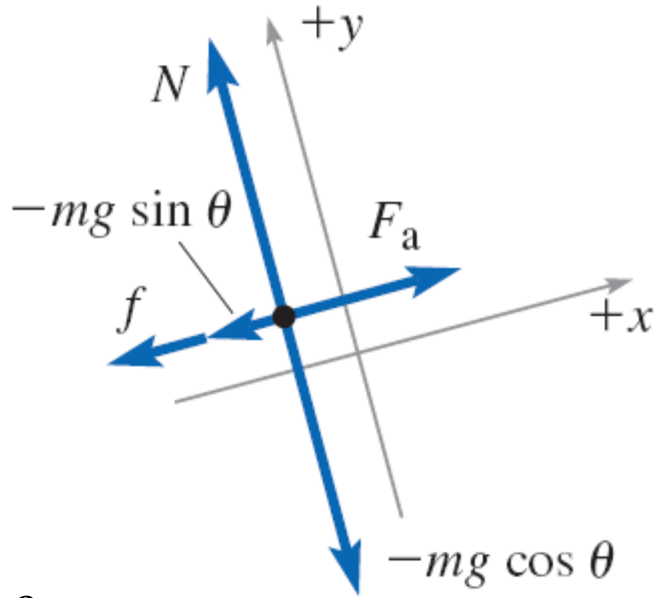

$$\sum F_x = F_{ax} + f_x + W_x + N_x = 0$$

$$F_{ax} = +F_a$$

$$f_x = -f_{s,max} = -\mu_s N = -\mu_s mg \cos \theta.$$

$$W_x = -mg \sin \theta \text{ and } N_x = 0$$

$$\sum F_x = F_a - \mu_s mg \cos \theta - mg \sin \theta + 0 = 0$$




$$\sum F_y = 0 = F_{ay} + f_y + W_y + N_y$$

$$F_{ay} = f_y = 0$$

$$W_y + N_y = 0$$

$$-mg\cos\theta + N = 0 \rightarrow N = mg\cos\theta$$

## Solution

$$\sum F_x = F_a - \mu_s mg \cos\theta - mg \sin\theta + 0 = 0$$

Solving for $F_a$,

$$F_a = mg\,(\mu_s \cos\theta + \sin\theta)$$

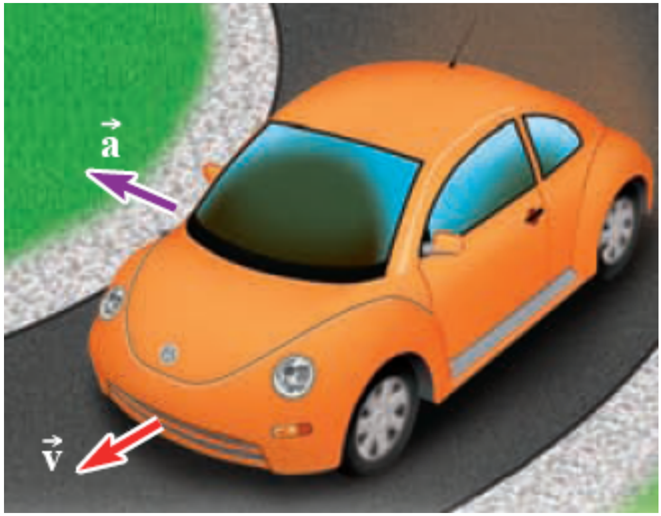
$$= 510 \text{ kg} \times 9.80 \text{ m/s}^2 \times (0.42 \times \cos 15° + \sin 15°)$$

$$= 3300 \text{ N}$$

An applied force that *exceeds* 3300 N starts the box moving up the incline.

**Unbanked Curves**

When you drive an automobile in a circular path along an unbanked roadway, friction acting on the tires due to the pavement acts to keep the automobile moving in a curved path.

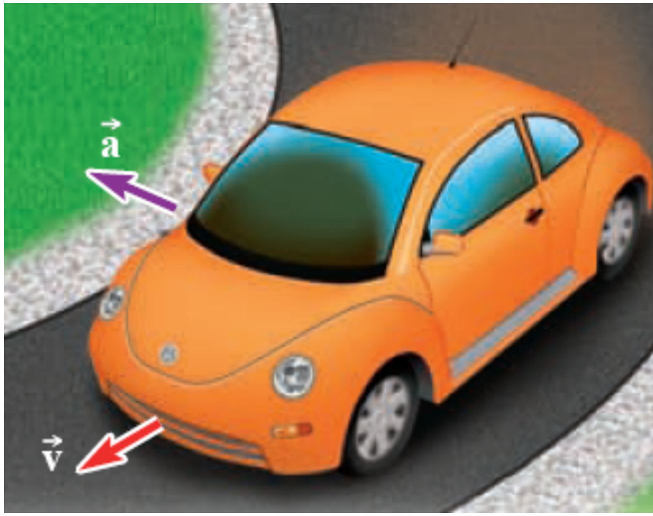



This frictional force acts *sideways*, toward the center of the car’s circular path.

The frictional force might also have a tangential component; for example, if the car is braking.

## Unbanked Curves

For now, assume that the car's speed is constant and that the forward or backward component of the frictional force is negligibly small.



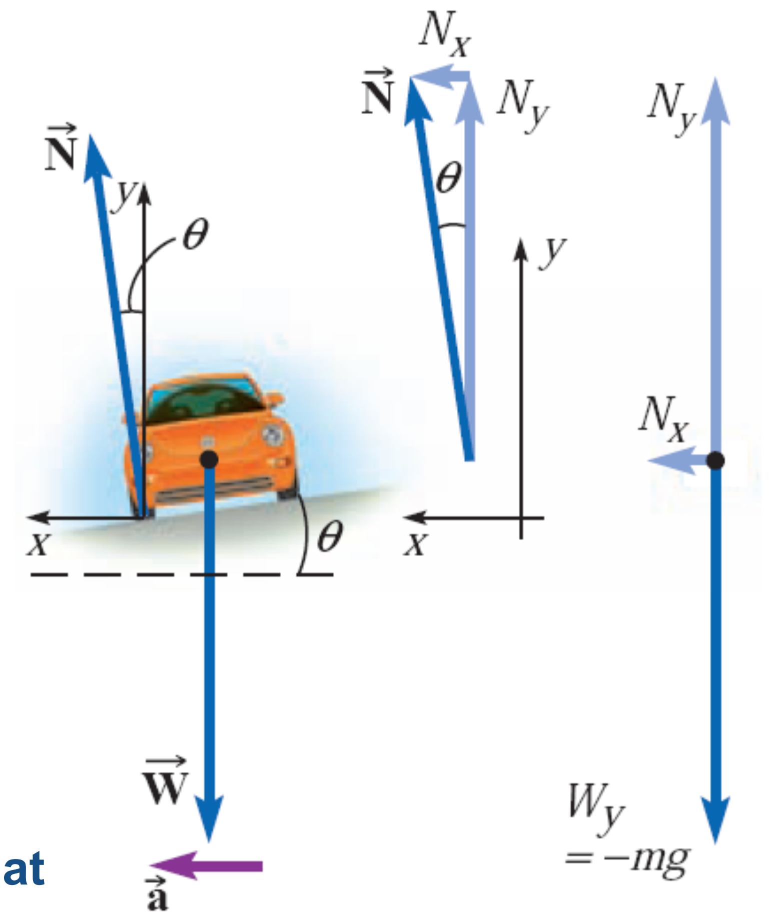

As long as the tires roll without slipping, there is no relative motion between the bottom of the tires and the road, so it is the force of ***static*** friction that acts to keep the car in the circular path

**Confusing! Let's pause to think about this one.**

**What does it mean if we have kinetic friction on the tire?**

# Application of Radial Acceleration and Contact Forces: Banked Curves

To help prevent cars from going into a skid or losing control, the roadway is often banked (tilted at a slight angle) around curves.




**Why is the acceleration in that direction? What does that mean?**

A car is going around an unbanked curve at the recommended speed of 11 m/s.

(a) If the radius of curvature of the path is 25 m and the coefficient of static friction between the rubber and the road is $\mu_s = 0.70$, does the car skid as it goes around the curve?

(b) What happens if the driver ignores the highway speed limit sign and travels at 18 m/s?

(c) What speed is safe for traveling around the curve if the road surface is wet from a recent rainstorm and the coefficient of static friction between the wet road and the rubber tires is $\mu_s = 0.50$?

(d) For a car to safely negotiate the curve in icy conditions at a speed of 13 m/s, what banking angle would be required?

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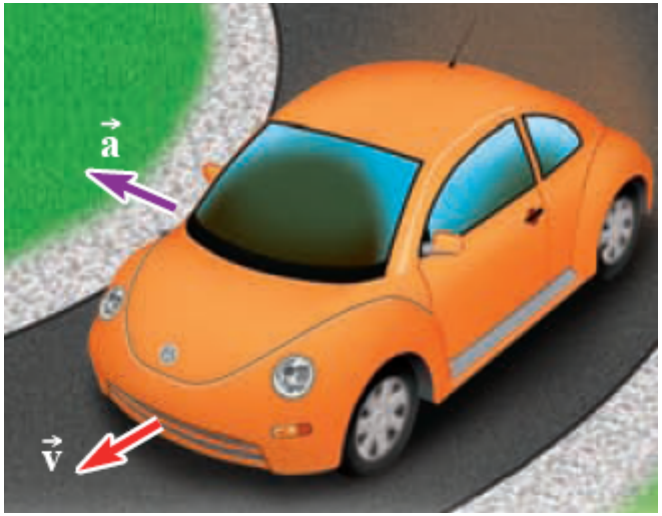
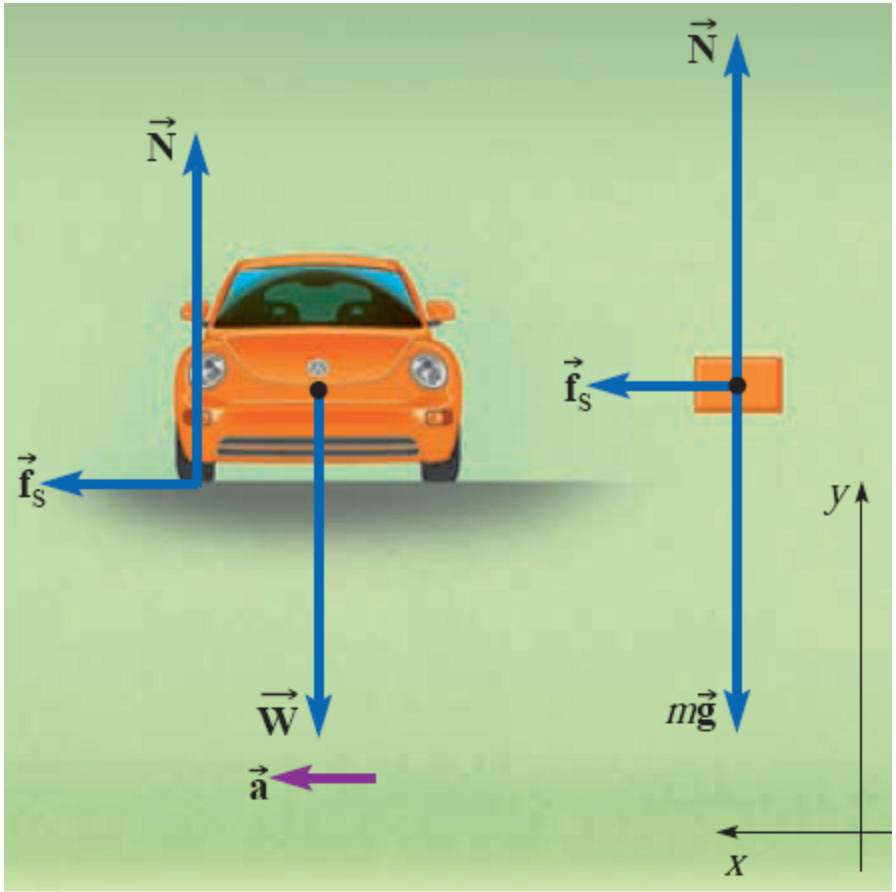


**Solution**

(a)

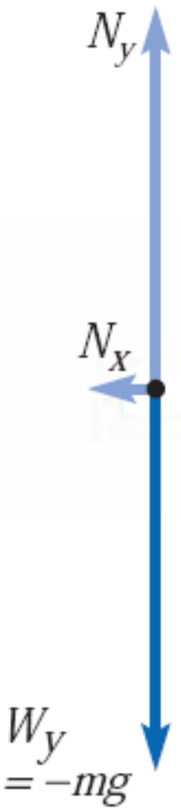
$$a_r = \frac{v^2}{r} = \frac{(11\ \mathrm{m/s})^2}{25\ \mathrm{m}} = 4.8\ \mathrm{m/s^2}$$

$$\sum F_r = ma_r = m\frac{v^2}{r}$$

$$\sum F_r = f_s = m\frac{v^2}{r}$$

$$f_s \le \mu_s N$$

$$\cancel{m}\frac{v^2}{r} \le \mu_s \cancel{m} g$$




Thus, the radial acceleration cannot exceed $\mu_s g$ .

**Inequality is good! We want to have different forces moving us in a circle depending on the speed and radius - friction can change (up to a maximum value)**

**Solution**

(a)

$$v \leq \sqrt{\mu_s g r}$$

$$v \leq \sqrt{0.70 \times 9.80\ \mathrm{m/s^2} \times 25\ \mathrm{m}} = 13\ \mathrm{m/s}$$

Since 11 m/s is less than the maximum safe speed of 13 m/s, the car safely negotiates the curve.

(b)

At 18 m/s, the car moves at a speed higher than the maximum safe speed of 13 m/s. The frictional force cannot supply the radial acceleration needed for the car to go around the curve—the car goes into a skid.

# Solution

(c)

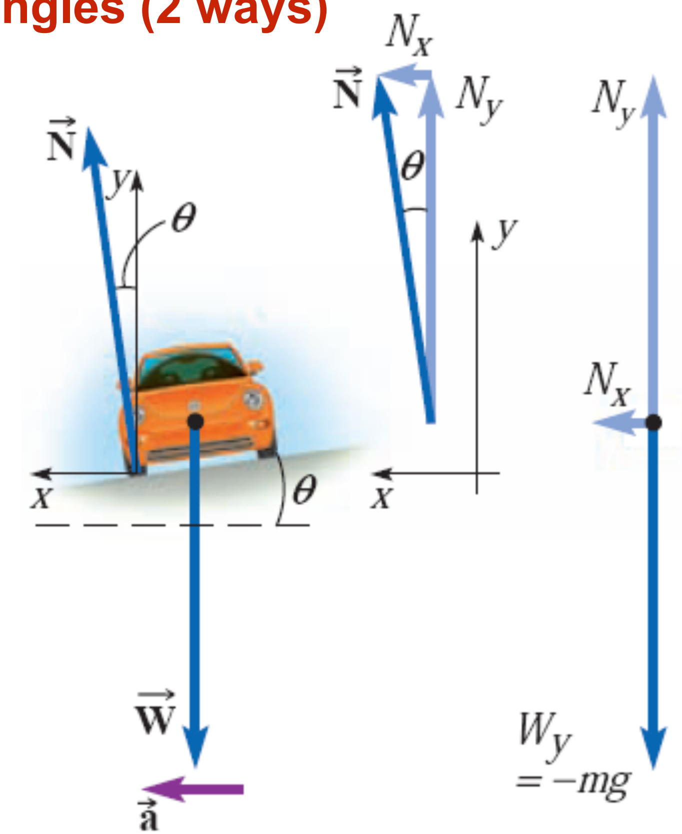
$$v \le \sqrt{\mu_s g r}$$

$$\mu_s = 0.50$$

$$v_{\max} = \sqrt{\mu_s g r} = \sqrt{0.50 \times 9.80\ \mathrm{m/s^2} \times 25\ \mathrm{m}} = 11\ \mathrm{m/s}$$

# Solution
(d)

Let's take a look at angles (2 ways)




Icy conditions = don't assume any frictional force!

$$\sum F_x = N \sin\theta = mv^2/r$$

$$\sum F_y = N\cos\theta - mg = 0$$

$$\frac{N\sin\theta}{N\cos\theta} = \tan\theta = \frac{mv^2/r}{mg} = \frac{v^2}{rg}$$

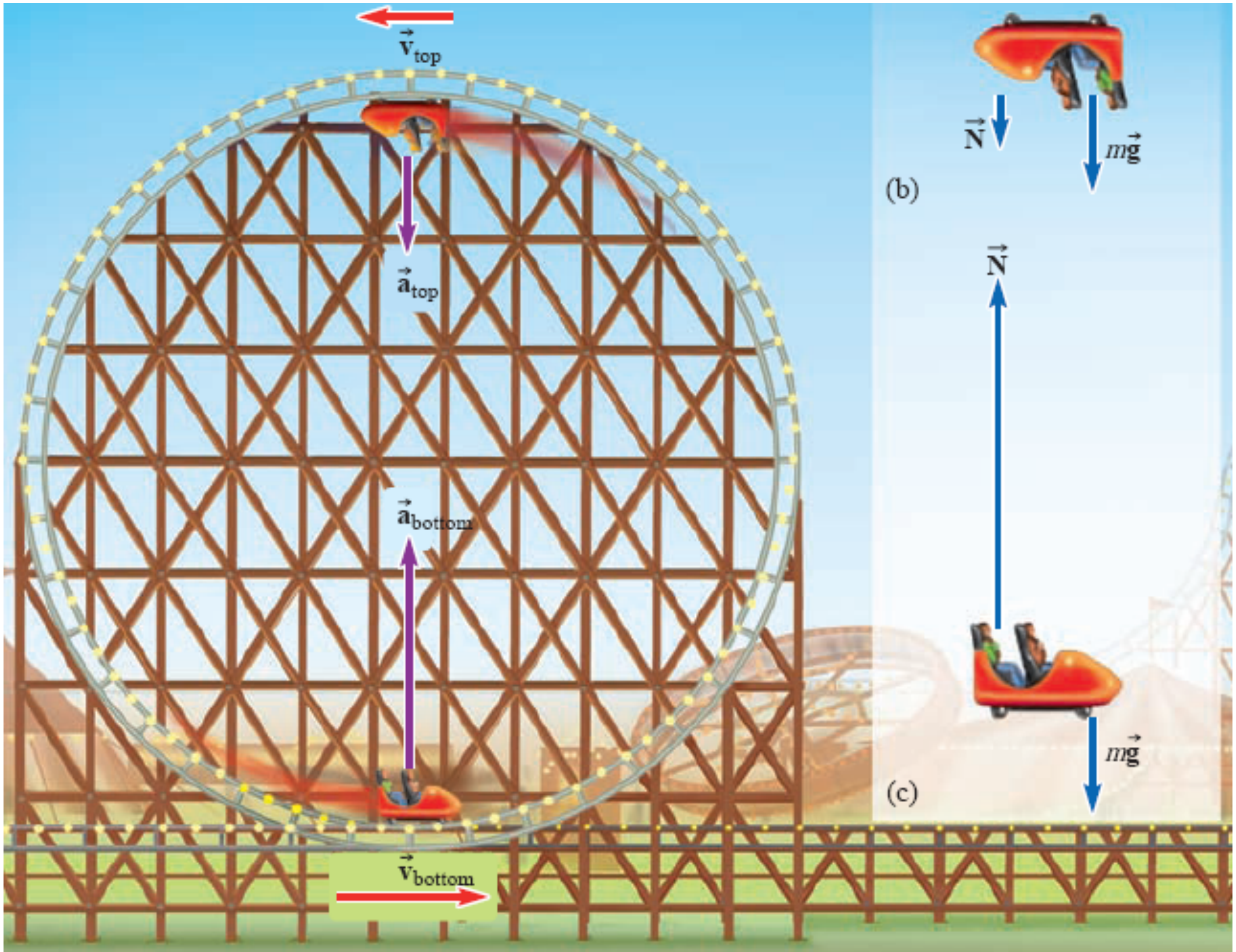
$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{(13\text{ m/s})^2}{25\text{ m}\times 9.80\text{ m/s}^2} = 35^\circ$$

## A more fun problem…

A roller coaster includes a vertical circular loop of radius 20.0 m.

What is the minimum speed at which the car must move at the top of the loop so that it doesn't lose contact with the track?

# A picture




**Why are normal forces different for top and bottom?**

# Solution

$$\sum F_r = N + mg = ma_r = \frac{mv_{\text{top}}^2}{r}$$

**Normal force can't be less than zero!**

$$N = \frac{mv_{\text{top}}^2}{r} - mg$$

$$m\left(\frac{v_{\text{top}}^2}{r} - g\right) \geq 0$$

$$v_{\text{top}} \geq \sqrt{gr}$$

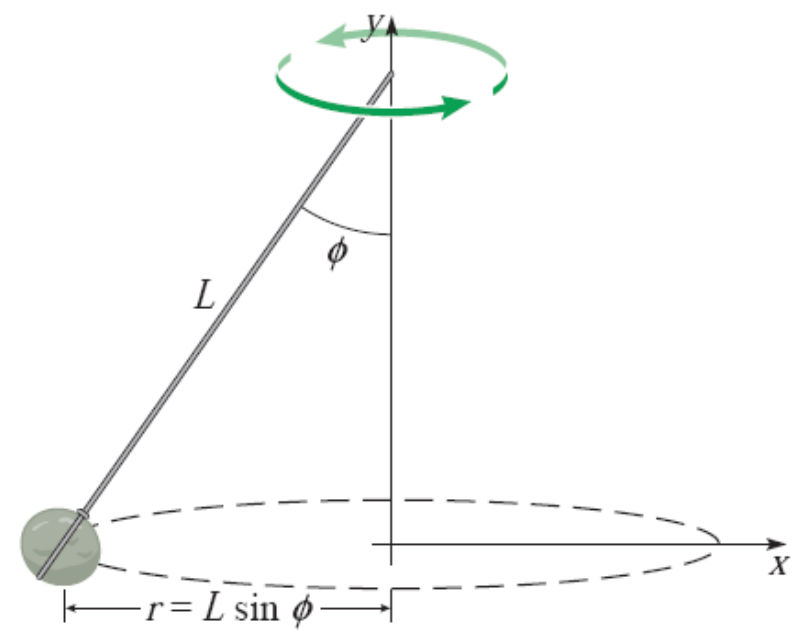
$$v_{\text{top}} = \sqrt{gr} = \sqrt{9.80\ \text{m/s}^2 \times 20.0\ \text{m}} = 14.0\ \text{m/s}$$

Suppose you whirl a stone in a horizontal circle at a slow speed so that the weight of the stone is *not* negligible compared with the tension in the cord. Then the cord cannot be horizontal—the tension must have a vertical component to cancel the weight and leave a horizontal net force.

If the cord has length $L$, the stone has mass $m$, and the cord makes an angle $\phi$ with the vertical direction, what is the constant angular speed of the stone?

**Stepping back** The net force must point toward the center of the circle, since the stone is in uniform circular motion.

With the stone in the position depicted in the figure, the direction of the net force is along the +*x*-axis. This time the tension in the cord does not pull toward the center, But the *net* force does.



# Solution

$$\sum F_x = T\sin\phi = ma_x = m\omega^2 r$$

$$r = L\sin\phi$$

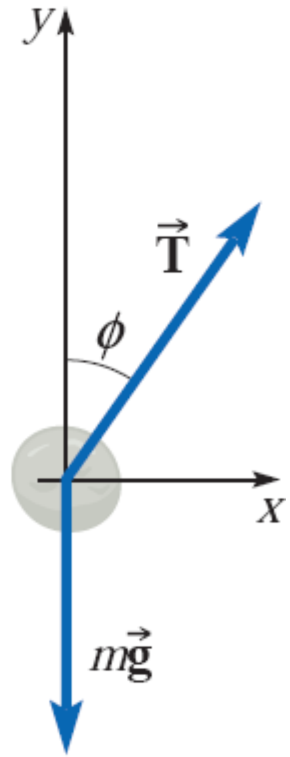
$$\sum F_x = T\cancel{\sin\phi} = m\omega^2 L\cancel{\sin\phi}$$

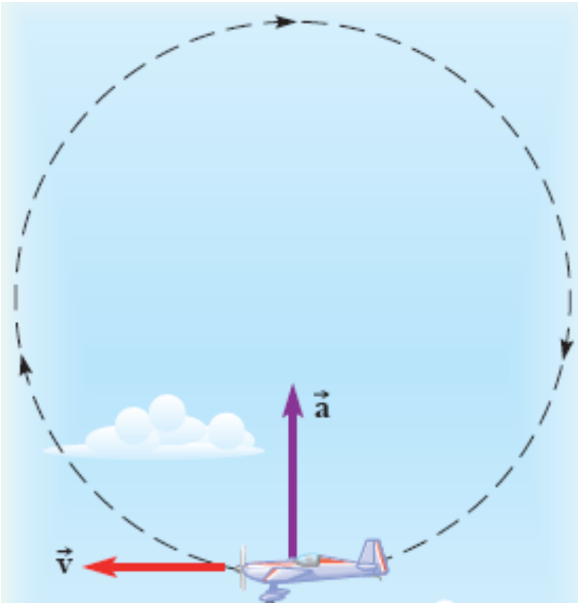
$$T = m\omega^2 L$$

$$\sum F_y = T\cos\phi - mg = ma_y = 0 \quad\Rightarrow\quad T\cos\phi = mg$$

$$(m\omega^2 L)\cos\phi = mg$$

$$|\omega| = \sqrt{\frac{g}{L\cos\phi}}$$






Dave wants to practice vertical circles for a flying show exhibition.

(a) What must the minimum radius of the circle be to ensure that his acceleration at the bottom does not exceed 3.0 *g* ? The speed of the plane is 78 m/s at the bottom of the circle.

(b) What is Dave's apparent weight at the bottom of the circular path? Express your answer in terms of his true weight.

**Strategy**

For the *minimum* radius, we use the maximum possible radial acceleration since $a_{\mathrm{r}} = v^2/r$

**Smallest radius = largest acceleration here!**

So….

$$r = \frac{v^2}{a_{\mathrm{r}}} = \frac{v^2}{3.0g}$$

$$= \frac{(78\ \mathrm{m/s})^2}{3.0 \times 9.8\ \mathrm{m/s^2}} = 210\ \mathrm{m}$$

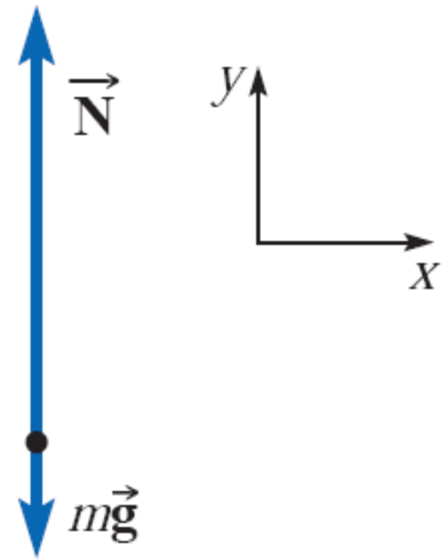
**Solution**

(b)

$$\sum F_y = N - mg = ma_y$$

$$a_y = +3.0g$$

$$W' = N = m(g + a_y) = 4.0mg$$




His apparent weight is 4.0 times his true weight.

## Air resistance and friction in fluids

The friction and air resistance in a fluid (air is a gas is a fluid) is a **force (drag force)**, with direction that is for our purposes always in a direction that opposes the current motion (**v**) of an object.
In other words, it is a vector pointing in the -**v** direction.

For example, if an ball moves through water or a particle moves through air in the (+x) direction, the air resistance or frictional force is in the (-x) direction.

**From experience, do we expect the drag force to be large or small for large |v|?**
**What happens if v=0 (object at rest)?**

## Air resistance and friction in fluids

How to think about drag forces? As an object moves through the fluid, it knocks into particles (water or air molecules, for examples). It must push them out of the way to move forwards, but in doing so, these molecules push backwards on the object

## Air resistance and friction in fluids

For small speeds, the magnitude of the drag force is proportional to v. In that case, $\mathbf{F} = -b\mathbf{v}$ (why the minus sign?), where b is a constant of proportionality to be determined

For larger speeds and in most cases of interest to us, it is typically proportional to $v^2$. In this case, the larger the cross-sectional area perpendicular to the direction of motion, the larger the drag force **(why?).** The larger the density of the fluid, the larger the drag force **(why?).** In these cases, then $F = (1/2)\, C\rho A v^2$, where C is a constant to be determined.

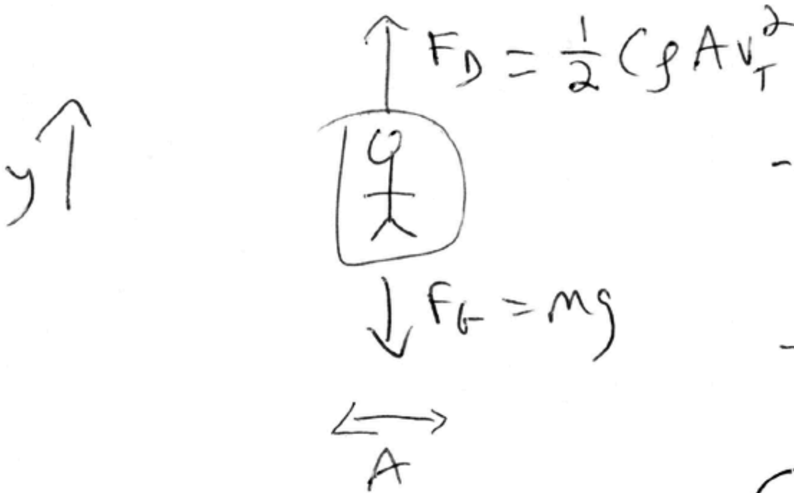
## Terminal speed

If an object is dropped from a large height, assuming that the drag force is proportional to $v^2$, what are the only forces on the object?

What happens if the object starts at rest? What happens as it picks up speed?

What happens if it travels for a long enough period of time? Let's work out together to find an object's **terminal speed**, and let's discuss what this means

As the object's speed increases, the drag force increases to oppose motion! At terminal speed, the speed is constant (so is the velocity!), and the **net force is zero.** The only forces acting on the object are the drag force and gravity




$$\rightarrow \sum F_y = \frac{1}{2} C\rho A v_T^2 - mg = 0$$

$$\rightarrow \frac{1}{2} C\rho A v_T^2 = mg$$

$$C\rho A v_T^2 = 2mg$$

$$v_T^2 = \frac{2mg}{C\rho A} \rightarrow \boxed{v_T = \sqrt{\frac{2mg}{C\rho A}}}$$

## What happens in a vacuum?

https://www.youtube.com/watch?v=_XJcZ-KoL9o

Time for group work!

https://forms.gle/Lhe2ETvrxj5QMjJo7


The maximum acceleration a pilot can be subjected to without losing consciousness is ~5.0g if the axis of acceleration is aligned with the spine. A pilot can avoid blackout up to ~9.0g by wearing special “g-suits” that help keep blood pressure in the brain at a sufficient volume

a) Assuming no g-suit, what is the minimum safe radius of curvature for a pilot in an F-15 in a horizontal circular loop at 750 km/h?

b) What is this radius if she is wearing the special suit?

A car approaches the top of a hill that is shaped like a vertical circle with a radius of 55.0m. What is the fastest speed that the car can go over the hill without losing contact with the ground?

A highway curve has a radius of 825 m. At what angle should the road be banked so that a car traveling at 26.8 m/s (60 miles per hour) has no tendency to skid sideways on the road (in other words, the frictional force is zero)?

An airplane is flying at constant speed $v$ in a horizontal circle of radius $r$. The lift force on the wings due to the air is perpendicular to the wings. At what angle to the vertical must the wings be banked to fly in this circle?

If a clothes washer's drum has a radius of 25 cm and spins at 4.0 revolutions per second, what is the strength of the artificial gravity to which clothes are subjected? Express your answer as a multiple of g

You place a block with a mass of 3.2 kg on a ramp. If the coefficient of static friction between the block and ramp is 0.3, what maximum angle can the ramp make with the horizontal before the block starts to slip down?

A sky-diver jumps out of an airplane and reaches terminal speed. What happens if she tucks in her arms and points herself downwards, reducing her cross-sectional area by a factor of 3?

You and your friend jump out of an airplane and each reach terminal speed. You have the same cross-sectional area, but you have a mass 50% larger than your friend. By what factor is your terminal speed larger than your friend's?

An ant is sitting 45 mm from the center of a DVD that is 120 mm in radius. The DVD speeds up from rest, and the ant is ejected when the DVD is rotating at 90 revolutions per minute. What is the coefficient of static friction between the ant and the surface of the DVD?

Why do you crumble up a sheet of paper before you toss it towards the garbage can?

Why do you crumble up a sheet of paper before you toss it towards the garbage can?

$V \propto \dfrac{1}{\sqrt{A}}$ and if the paper is crumbled, A is reduced so the paper has higher terminal velocity. That means less time and chance for air currents to cause you to miss!

You are riding in a car around a turn. You do not move with respect to the seat. The turn is flat.

a) Draw a FBD for you in the seat
b) Which force or forces are responsible for the centripetal force on you?
c) What happens to you if the driver takes the turn too quickly?

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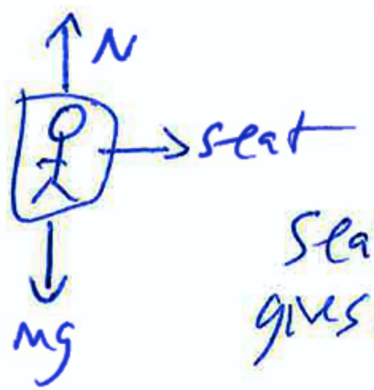
a) + b)




→ a

Seat pushing on you gives centripetal force!

c) If turn too quickly, you get pushed to one side of the car because seat can't provide enough force

A heavy box of tools rests on a tabletop. You push on the box with a horizontal force of 100 N and observe that the box does not move. You then push on the box with a horizontal force of 200 N and observe that the box still does not move. How does the magnitude of the static friction force compare to the force with which you push in each case?

A box is at rest with respect to the surface of a flatbed truck. The coefficient of static friction between the box and the surface is $\mu s$.

a) Find an expression for the maximum acceleration of the truck so that the box remains at rest with respect to the truck. Your expression should be in terms of $\mu s$ and g.
b) How does your answer change if the mass of the box is doubled?

Rochelle holds her 2.8 kg physics textbook by pressing horizontally against both side of the book with the palms of her hands. If the coefficient of static friction between her hands and the book is 0.50, what is the minimum force with which she must compress the textbook with each hand to keep it from slipping out?

A block with mass M = 2.0 kg is placed on an inclined plane. The plane makes an angle of 30 degrees with the horizontal, and the coefficients of kinetic and static friction between the block and the plane are $\mu_k$=0.400 and $\mu_s$ = 0.60. Will the block slide down the plane, or will it remain motionless? Justify your answer.

You pull a box with mass 5.5 kg up a ramp with angle 20 degrees from the horizontal by applying a constant 190 N force along the ramp. The coefficient of kinetic friction of the ramp is 0.15. Ignore air resistance. If the box starts out moving up the ramp at 0.3 m/s, how long does it take to pull the box up the length of the full ramp, 3.2 meters?